A trade algorithm for multi-region models subject to spillover externalities

Marian Leimbach,* Klaus Eisenack

January 12, 2007

*PIK - Potsdam Institute for Climate Impact Research, P.O. Box 60 12 03, D-14412 Potsdam, Germany, Tel. ++49/331/288-2566, e-mail: leimbach@pik-potsdam.de

Abstract

Within this paper, we present an algorithm that deals with trade interactions within a multi-region model. In contrast to traditional approaches, however, this algorithm is able to handle spillover externalities as well. We focus on technological spillovers which are due to capital trade. The algorithm of finding a pareto-optimal solution in an intertemporal framework is embedded in a decomposed optimization process. This paper analyzes convergence and equilibrium properties of this algorithm analytically as well as numerically. The spillover effect is quite difficult to capture analytically and even numerically challenging. Differences in the solutions of the Social Planner model and the decentralized model become apparent. But numerical evidence can be given that even in this case an equilibrium solution can be found. In the final part of the paper, we apply the algorithm to investigate possible impacts of technological spillovers. While benefits of technological spillovers are significant for the capital-importing region, benefits for the capital-exporting region depend on the type of regional disparities and the resulting specialization and terms-of-trade effects.

JEL classification: O41;F17;F43;C68;O39
keywords: multi-region modeling, technological spillovers, externality, general equilibrium, decentralized optimization

1 Introduction

In a multi-region setting, investment and trade decisions of each region depend on decisions of each other region. Multi-region modeling becomes a challenging task when different regional interactions are considered. In classical economics and trade theory, prices are the major tool for coordinating regional interactions (cf. Samuelson (1952), Negishi (1972)). Price-based adjustment algorithms like the standard Walrasian excess demand algorithm can be used for finding equilibrium prices. Early work on algorithms that help to find equilibrium prices numerically was summarized by Scarf (1984). More recently, e.g. Kumar and Shubik (2004) and Luenberger and Maxfield (1995) presented advanced algorithms for the computation of competitive equilibria.

Additional challenges arise from the existence of externalities (Farmer and Lahiri, 2005; Greiner and Semmler, 2002) and the fact that international trade is an inherent dynamic process (Oniki and Uzawa, 1965; Stiglitz, 1970). Our motivation in developing and applying a new method of multi-region modeling is due to the deficits of traditional approaches in dealing with externalities, in particular with technological spillovers. Within this study, the reference point is the Negishi approach - a well-known solution technique for multi-region modeling (Manne and Rutherford (1994), Leimbach and Toth (2003)). Technically, the Negishi approach merges the regions' optimization problems under a global welfare function. It therefore differs from approaches based on decentralized decision-making. Essentially, it internalizes the coordination function which in decentralized models is played by the market or a virtual auctioneer.

Some well-known models in climate economics, e.g. MERGE (Manne et al. (1995), Manne and Richels (1995)) and RICE (Nordhaus and Yang (1996)), applied the traditional Negishi algorithm in order to find a general equilibrium in an intertemporal optimizing framework. The Negishi approach is numerically quite efficient and in cases without externalities, where the social optimum corresponds to the competitive market equilibrium, often superior to the Open-Nash-loop approach in finding a market solution. However, the Negishi model has to be modified in order to incorporate spillover externalities. Yet, as Leimbach and Edenhofer (2007) have shown, this modified Negishi approach is limited in capturing the external effects induced by technological spillovers.

Following the basic idea of Leimbach and Edenhofer (2007), we present an alternative trade algorithm that is able to internalize the external spillover effects completely. The concept of spillovers applied here is different from the spillover concept used in other studies (e.g. Corsetti et al. (2005), Böhringer and Rutherford (2002)) where spillovers represent secondary effects of relative price changes and exogenous productivity changes. In this study, technological spillovers refer to situations where the presence of physical capital, produced abroad, affects efficiency or productivity levels of domestic firms in a host economy. Spillovers of such a kind represent an external effect. Yet, they have certainly an impact on the investment decisions of the regional agents. This feedback is widely neglected in existing models. This is due to the increasing returns to scale effect which defies control of classical general equilibrium theory. Moreover, if empirical studies suggest a link between positive productivity gains and foreign investments, why should agents not take this into account in decision-making and why should foresighted agents not be more proactive in attracting foreign direct investments and capital exports? In contrast to existing intertemporally optimizing models, we present an approach where technological spillovers are anticipated by the regional actors, hence, influencing the dynamics of the control variables.

The body of empirical research on spillover externalities has grown rapidly. A majority of studies indicate positive spillover effects from foreign investments (e.g. Kokko (1993), Takii (2004), Jordaan (2005)). Takii (2004) demonstrated for several countries that foreign firms (resulting from foreign direct investments) tend to have higher productivity than domestic ones, hence improving the host country's aggregated productivity. Likewise, empirical results presented by Lee (1995) imply that imported capital goods have a much higher productivity than domestically-produced capital goods.

Within the model presented in the next section, technological spillovers are attributed to capital export rather than to foreign direct investments. However, the spillover mechanism for both types of transfer of physical capital is thought to be nearly the same, since technological know-how is embodied in the machinery that is build up abroad in either way.

The paper is structured as follows: The trade algorithm is elaborated in combination with a stylized multi-region model, the mathematical structure of which is presented in section 2. In order to make this model computable, we carry out a decomposition. The decomposed model and the iterative algorithm that searches for a pareto-optimal solution are presented in section 3. We discuss the equilibrium properties of such solution in section 4. While analytical evidence can be provided for the case without spillover externality, numerical experiments support the reliability of the trade algorithm for spillover cases. Results from numerical experiments on potential welfare and terms-of-trade implications of technological spillovers are presented in section 5. It turns out, that at least for the capital-exporting region benefits from spillover are sensitive to the type of specialisation and that taking technological spillover into account could change the optimal trade pattern. We end with some conclusions in section 6.

2 The multi-region model

In this section, we present the multi-region model. It represents a dynamic model of international trade. The decomposition that makes the model computable is given in the next section. We decided against a presentation of a more general model that differs from the computational model in order to avoid redundancy later on. The algorithm that solves the model, however, can be applied to other specifications (in particular of the production functions) as well. Analytical analysis, provided in the Appendix A, is based on a more compact exposition of the model. The model can be classified as an optimal economic growth model. A representative agent is assumed to summarize households' consumption decisions and firms' investment and trade decisions.

The following indices are used throughout the model presentation:

t	1,2,...,T	time periods,
i, k	1,2,...,n	regions,
j		goods,
r	1,2,...,m	iterations.

With $J = \{G, F\}$ and $j \in J$, the following types of trading goods are distinguished:

G	consumption good,
F	investment good.

Each good is produced in a different sector. Hence, j also represents a sectoral index. I denote the sectoral index by a superscript throughout the model presentation. Although the model and the equations are implemented in a time-discrete framework, I use the continuous formulation throughout the paper for simplicity. Time derivatives are represented as usual. Each variable actually bears the time and iteration index. For transparency reasons, they are suppressed as often as possible.

The objective of the multi-region model is to maximize the welfare U of n regions:

$$\max\{U\} \qquad U = (U_1, U_2, ..., U_i, ..., U_n) \tag{1}$$

with

$$U_i = \int_{t=1}^{T} f[C_i(t)] \cdot e^{-\rho t} dt. \tag{2}$$

These welfare functions measure the utility of each regions' representative household. Utility is expressed as a function *f* of the consumption path *C* subject to discounting by the pure rate of time preference ρ. As an instance of the function *f*, I apply a common logarithmic or Bernoullian utility function:

$$f(C_i) = L_i \cdot ln \frac{C_i}{L_i} \tag{3}$$

where *L* represents the regions' population which provides the exogenously given production factor labor. The production function Y^G for the consumption goods sector is specified as a Cobb-Douglas function :

$$Y_i^G = A_i \cdot [(1 - \theta_i) \cdot K_i]^{\alpha_i} \cdot L_i^{1-\alpha_i} \tag{4}$$

with

$$0 < \theta_i < 1. \tag{5}$$

Variable *A* denotes the productivity level and variable θ denotes the share of total capital stock which is allocated to the investment goods sector. *K* and *L*, respectively, represent the capital and labor production factors; α is the capital-output elasticity parameter. We assume that labor is used only in the consumption goods sector. This means that there is a fixed endowment of this production factor which, furthermore, is internationally immobile. This prevents the model from coming up with exaggerated specialization.

Investment goods production is assumed to be a function of capital only. Region i's production function of investment good F is given by

$$Y_i^F = \kappa_i \cdot (\theta_i \cdot K_i)^{\phi}. \tag{6}$$

When the elasticity parameter ϕ is equal to 1, this equation becomes a Leontief-type production function and parameter κ could be interpreted as a technological coefficient (investment goods output per unit capital stock). [1]

Capital is allocated from a common pool. Thus, perfect cross-sectoral mobility of capital is implicitly assumed (we neglect the vintage and putty-clay structure of the capital stock). Capital accumulation follows the standard equation of capital stock formation:

$$\dot{K}_i = I_i + \sum_{k=1}^{n} X_{ki}^F - \delta_i \cdot K_i. \tag{7}$$

I represents domestic investments, while X^F accounts for investment goods imports. δ is the depreciation rate of capital. For simplicity reasons, I assume perfect substitutability between domestic and imported capital goods. The same assumption applies to consumption goods. The output of the consumption goods sector represents the regional gross product net of investments. It is used to meet demands on consumption and exports, while being incremented by imports:

$$Y_i^G = C_i + \sum_{k=1}^{n} (X_{ik}^G - X_{ki}^G). \tag{8}$$

X_{ik} denotes the export from region i to region k. It simultaneously denotes the import of region k from region i. Note that the trade variables represent net export and net import values. The use of separate export and import variables is due to the subsequent modeling of technological spillovers.

[1]Following the neoclassical assumption of diminishing marginal productivity, we chose a value for ϕ that is slightly lower than 1. With $\phi < 1$, this production function exhibits decreasing returns to scale and the conventional remuneration of production factors in accordance with its marginal productivities is incomplete. Implicitly, there is a kind of monopoly rent that restricts further entry of firms. However, this does not limit the model's capability in finding an interregional competitive equilibrium, which is the focus here.

The investment goods sector provides domestic investments and meets foreign demands on investment goods:

$$Y_i^F = I_i + \sum_{k=1}^{n} X_{ik}^F. \tag{9}$$

The range of regional interactions usually modeled is extended by technological spillovers. Technological spillovers increase the host country's productivity through capital goods import. Within the model, an additional change of the total factor productivity A in a region i is a function of capital good exports X^F from region k to region i and of productivity differences between both regions:

$$\dot{A}_i = \sum_{k=1}^{n} \left[\left(\frac{X_{ki}^F}{K_i} \right)^{\zeta} \cdot \beta \cdot max(0, A_k - A_i) \right]. \tag{10}$$

Parameter β represents the spillover intensity, i.e. the actual impact of technological spillovers in the host country, while ζ $(0 < \zeta < 1)$ depicts the elasticity of productivity changes on capital goods imports. The capital import variable is divided by the capital stock in order to avoid scaling effects. By choosing control variable X^F, regions influence the extent of technological spillovers, i.e. technological spillovers are subject of rational expectations. Consequently, technological progress is partly endogenized.

An intertemporal budget constraint based on world market prices p^j has to be met by each region. It is given by

$$\int_{t=1}^{T} B_i(t)dt = 0 \tag{11}$$

where

$$B_i = \sum_{j \in J} \left(p^j \cdot \sum_{k=1}^{n} [X_{ik}^j - X_{ki}^j] \right). \tag{12}$$

This equation serves to level off the trade deficits of each region in the long run. The model is completed by initial conditions:

$$K_i(0) = k_i \tag{13}$$

$$A_i(0) = a_i \tag{14}$$

and non-negativity conditions:

$$C_i, A_i, I_i, K_i, Y_i^j, X_{ik}^j \geq 0. \tag{15}$$

3 Decomposed model

Because each region is represented by a distinct and separate utility function, the multi-region model is not operable offhand. Forming a global welfare function is a possible next step towards a solution. The Negishi model represents such a global welfare maximization problem. Negishi (1972) proved the correspondence between the international competitive equilibrium and a welfare optimum [2]

$$maxW(U(C_i)) = \sum_{i=1}^{n} \omega_i \cdot U_i(C_i) \tag{16}$$

for a particular set of strictly positive welfare weights ω_i with $\sum \omega_i = 1$ under usual convexity assumptions. Negishi presented a mapping to derive ω_i. This mapping has a fixed point.

However, the Negishi model fails when technological spillovers have to be considered. Leimbach and Edenhofer (2007) have shown that the Negishi algorithm can not consider spillover effects because it is not able to distinguish between export and import prices and quantities. To overcome this problem, we developed an alternative approach to multi-region modeling based on a decomposition of the original optimization problem into single regional optimization modules and a trade module. While the present decomposition resembles that of Leimbach and Edenhofer (2007), the resulting trade module is a completely different one.

3.1 Region module

In each region module, the welfare of the considered region only is maximized:

$$\max_{\theta_i} U_i. \tag{17}$$

This decentralized optimization problem is subject to the constraints (2)-(10) and (13)-(15) from the basic model in the previous section. Note that the intertem-

[2] Negishi (1972) characterized the solution as a maximum point of a social welfare function which is a linear combination of utility functions of individual consumers, with the weights in the combination in inverse proportion to the marginal utilities of income.

poral budget constraint is no longer part of the optimization problem. Instead, each region is restricted by import and export bounds $\bar{X}$. These bounds are primarily algorithmic devices generated by the trade module (see below). However, they can be conceived as the net import demand and export offer from other regions that the optimizing region expects to face. Each region can only import what all other regions offer to export to this region:

$$X^j_{ki,r} = \bar{X}^j_{ki,r-1}. \tag{18}$$

Analogously, each region has to meet imports that all other regions demand from this region, yielding the following export constraint:

$$X^j_{ik,r} = \bar{X}^j_{ik,r-1}. \tag{19}$$

Despite the fact that $\bar{X}$ represents the right hand side of an equation, we keep to refer to it as a bound.

3.2 Trade module

The purpose of the trade module is to determine the trade flow boundaries $\bar{X}$. To this end, the trade module is formulated as a single multi-region model. Equally to the Negishi approach, a global objective function combines the welfare functions of all regions by means of welfare weights:

$$\max_{\theta_i, \bar{X}^j_{ik}} W = \sum_{i=1}^{n} w_i \cdot U_i. \tag{20}$$

Moreover, the optimization problem includes constraints (2)-(10) and (13)-(15) for each region with the only difference being that the trade flow variables X are replaced by $\bar{X}$. In order to avoid artificial investment goods exports in anticipation of spillover gains from re-exports, an additional constraint states that each region can only be a net exporter or importer of investment goods:

$$\sum_{t=1}^{T}\sum_{k=1}^{n} \bar{X}^F_{ik}(t) \cdot \sum_{t=1}^{T}\sum_{k=1}^{n} \bar{X}^F_{ki}(t) = 0. \tag{21}$$

Indeterminacy could cause simultaneous goods export and import in a single region. This is prevented by:

$$\sum_{k=1}^{n} \bar{X}_{ik}^{G}(t) \cdot \sum_{k=1}^{n} \bar{X}_{ki}^{G}(t) = 0. \tag{22}$$

The trade module represents the problem as a Social Planner problem, generating a solution that assumes the decentralized actors to behave socially optimal.

3.3 Iterative trade algorithm

The decomposed model is solved iteratively by the following steps:

1. Fixing the trade structure.
2. Solve region modules (decentralized model).
3. Extract prices for intertemporal budget constraints.
4. Adjust welfare weights.
5. Solve trade module (Social Planner model) and derive export/import bounds.

In each iteration, the welfare weights of the objective function (20) are computed from the deviation of the intertemporal trade balance (intertemporal budget constraint):

$$w_{i,r+1} = w_{i,r} \cdot (1 + h(B_{i,r})). \tag{23}$$

The particular implementation of function *h* follows Leimbach and Toth (2003, p.163):

$$h(B_{i,r}) = \sum_{t=1}^{T} B_{i,r}(t) \cdot \left[\frac{(\gamma \cdot ln(r) + 2 \cdot \gamma)}{\sum_{k=1}^{n} V_k + V_i} \right] \tag{24}$$

where

$$V_i = \sum_{t=1}^{T} \left[p^G(t) \cdot C_i(t) + \sum_{k=1}^{n} p^j(t) \cdot (X_{ik}^j(t) - X_{ki}^j(t)) \right]. \tag{25}$$

Weighting factor *V* can be interpreted as the economic power of each region; γ is a parameter that facilitates the convergence process. The iterative procedure of adjusting the welfare weights assures that the intertemporal budget constraint is met.

Most crucially, computing *B* and *V* in eq. (24) demands for world market prices p^j. These prices are computed as weighted averages of the regional import prices *pi* and export prices *pe*:

$\boxed{\text{Mit Appendix abgleichen!}}$

$$p^j = \left[\frac{\sum_{i=1}^{n} \sum_{k=1}^{n} (pi_{ik}^j \cdot X_{ik}^j + pe_{ik}^j \cdot X_{ik}^j)}{\sum_{i=1}^{n} \sum_{k=1}^{n} X_{ik}^j} \right] / 2 \,, \qquad i \neq k. \tag{26}$$

Both prices *pi* and *pe* consist in shadow prices of constraints (18) and (19), which can be described in the form of partial derivatives (with U^* as maximum welfare in iteration r):

$$pi_{ki}^j = \frac{\partial U_i^*}{\partial \bar{X}_{ki}^j} \tag{27}$$

$$pe_{ik}^j = -\frac{\partial U_i^*}{\partial \bar{X}_{ik}^j}. \tag{28}$$

Import and export prices are specific for each time period and possibly differ from each other. Likewise, regional prices are not expected to necessarily converge offhand [3]. Convergence, however, can be monitored for the willingness to pay and the willingness to accept (see next section). As for the prices, the following relations hold:

- non-spillover case:

$$pe_{ik}^j \neq pi_{ik}^j \,, \quad i \neq k$$

$$pe_{ik}^j \neq pe_{ki}^j \,, \quad i \neq k$$

$$pi_{ik}^j \neq pi_{ki}^j \,, \quad i \neq k$$

$$pe_{ik}^j = pi_{ki}^j \,, \quad i \neq k$$

$$\forall i \quad pe_{il}^j = pe_{ik}^j \,, \quad l \neq k$$

[3]Non-convergence of shadow prices is on the one hand due the dimension of these shadow prices. They are measured in welfare units and due to the form of the welfare function, a further unit of a consumption good in a poor region might always have a higher shadow price than in a richer region. On the other hand, there is the impact of the intertemporal budget constraint which for regions with a current account deficit requests to export tradables. The designated exporter may have higher marginal utility (shadow prices) than the importing region with respect to the trading good. Each export of such a good, however, increases its shadow price and hence the difference to the respective price within the importing region.

$$\forall i \quad pi^j_{li} = pi^j_{ki} \ , \quad l \neq k$$

- deviations from above in spillover case:

$$\exists i, k \ \ pe^F_{ik} \neq pi^F_{ki} \ , \quad i \neq k$$

$$\exists i \ \ pi^F_{li} \neq pi^F_{ki} \ , \quad l \neq k.$$

In the presence of spillovers, export prices of investment goods do not, in general, correspond to import prices, and the investment good import prices may also differ depending on the region from which the capital good is imported.

The algorithm proposed in this paper couples the trade module and the regional modules for numerical experiments to compute a pareto-optimal solution. The trade module can be conceived as a virtual coordinator that has to compute an optimal allocation of all traded goods. Whereas the Walrasian auctioneer monitors excess of demand over supply (or vice versa) and adjusts prices based on this information, here the coordinator knows the willingness of the trading partners to pay for another unit of import or to accept another unit of export. Based on this, the allocation of exports and imports is adjusted. For the single region, this allocation appears as the foreign demand on and supply of trading goods.

Capturing interactions between the regions by means of the virtual coordinator is an iterative process. Within each iteration, first the region modules, confronted with new export/import bounds, are solved. The region modules provide shadow prices for export and import goods from each region, which are used by the trade module to update export/import bounds. Figure 1 shows the data flow between the modules. This iterative adjustment process ends when it converges, implying that the trade structure does not change anymore, i.e. for all export variables and prices it holds that

$$\forall j : |\bar{X}^j_r - \bar{X}^j_{r-1}| \leq \epsilon, |p^j_r - p^j_{r-1}| \leq \epsilon \tag{29}$$

and the algorithm converges in the sense that

$$\forall i : |w_{i,r} - w_{i,r-1}| \leq \epsilon \tag{30}$$

and

$$\forall i : |\bar{C}_i - C_i| \leq \epsilon, |\bar{\theta}_i - \theta_i| \leq \epsilon, \tag{31}$$

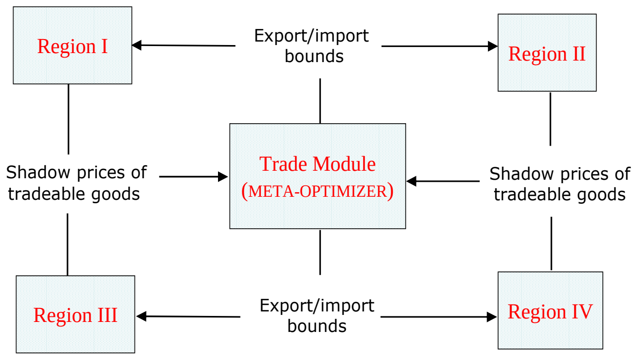

Figure 1: Data flow between modules

where $\bar{C}_i, \bar{\theta}_i$ denote consumption and capital allocation computed by the trade module, while C_i, θ_i the analogue quantities computed by the regional modules.

It should be noted that the present algorithm operates in an intertemporal model setting which may include transitional dynamics. Figure 2 demonstrates the convergence process for the time trajectory of the consumption good export variable. It results from a two-region setting as analyzed within the model experiments presented in the next section.

4 Equilibrium solution

Does the solution of the decomposed model represent an international competitive equilibrium? Based on Ginsburgh and Waelbroeck (1981, p.10), we define an international competitive equilibrium as the allocation paths θ_i, C_i, and $X^j_{i,k}$, $i, k = 1, 2, ..., n$, $j = 1, 2, ..., l$, supported by non-negative price vectors p^j, such that the following conditions hold:

(a) equality of exports and imports for traded goods on the international market,

(b) utility maximization by each country: θ_i, C_i, and X^j_i maximize U_i subject

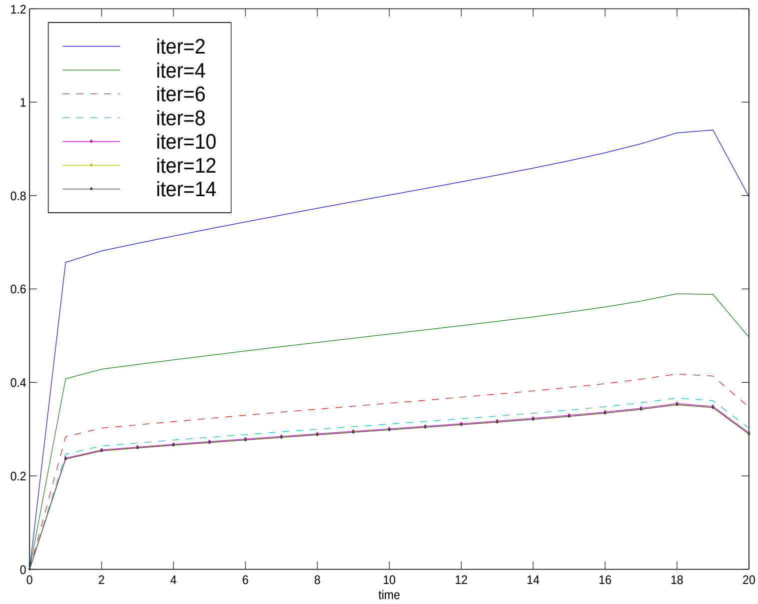

Figure 2: Convergence of consumption good export bound

to the following constraints:

(b1) feasibility of produced quantities ($Y_i^j \le Y_i^j(\theta_i)$),

(b2) intertemporal budget constraint ($B_i(T) = 0$),

(b3) availability of quantities on the domestic market ($I_i \ge 0, C_i \ge 0$).

To show this one has to prove that the result of a convergent trade algorithm fulfills these criteria. This is necessary since the introduction of the export and import bounds (eq. 18 and eq. 19) in the regional module of the decomposed model is in contrast to the maximization problem as described in the definition of the competitive equilibrium where imports and exports can be freely chosen to optimize utility. Therefore, verifying that the trade algorithm computes an equilibrium solution requests for mathematically proving that the export/import quantities and prices determined by the trade module indeed respect conditions (a) and (b).

Please note that this is not a corollary of Negishi's theorems due to the following reason: Negishi considers trade in consumption goods only and does not separate import and export quantities and prices. Capital goods trade and distinct export and import prices are, however, necessary to consider technological spillovers.

We now demonstrate the required properties for the case without spillover externalities (Appendix A provides a concise analytical deduction).

Condition (a) is obviously met by definition of the import/export variables.

Condition (b) requires to show that for the prices p^F, p^G entailed by the result of the iterative algorithm (which is a fixed point of the composition of the trade and the regional module), the regions in the basic decentralized model, which is not restricted by trade bounds but an intertemporal budget constraint, choose exactly the same imports and exports as in the fixed point to obtain maximal welfare. In other words, the optimal regional trade quantities equal the convergence result of the iterative trade algorithm.

From the first order conditions *of the trade module* it can be derived that

$$\forall i, k = 1, \ldots, n : \frac{C_i}{C_k} = \frac{w_i}{w_k} \tag{32}$$

and

$$\forall i, k = 1, \ldots, n : \frac{\bar{\lambda}_k}{\bar{\lambda}_i} = \frac{w_i}{w_k}, \tag{33}$$

where $\bar{\lambda}$ represents the shadow price of capital.

If the algorithm converges, the intertemporal budget balance is met and θ_i and C_i are identical in the regional and the trade module.

It remains to show that these results indeed meet the first order conditions of the optimization problem of the basic model. In this model, the budget balance has to considered explicitly by introducing new costate variables π_i which appear to evolve according to $\pi_i(t) = \pi_i(0)e^{\rho t}$. Capital and consumption trade is then choosen such that

$$\pi_i p^G = \frac{1}{C_i}, \tag{34}$$

and

$$\pi_i p^F = \hat{\lambda}_i, \tag{35}$$

where $\hat{\lambda}_i$ denote the shadow prices of capital in the basic model. This simply means that prices equal marginal utility, corrected by a factor representing the budget constraint. When selecting $\pi_i(0) = w_i^{-1}$, it can be shown that this implies the same values for the control variables as the trade algorithm implies. This equality is an intertemporal analogue to the Negishi approach, where the inverse of Negishi weights is equal to the marginal utilities of income.

We have thus shown that the trade algorithm computes an international competitive equilibrium in the non-spillover case, if it converges. A proof of convergence is beyond the scope of this paper. We instead provide numerical evidence. However, the proof of existence given in Appendix A and summarized here provides some further interesting properties of the equilibrium solution.

Alternatively, to verify whether the numerically approximated fixed point indeed represents an international competitive equilibria, a numerical test can be performed for each experiment. After the trade algorithm has finished, for each region the optimization problem eq. (17) s.t. (3-15), (21), (22) is solved numerically. This is a standard decentralized model. The export/import bounds no longer apply, but the intertemporal budget constraint is re-introduced with the prices p^j computed by the trade algorithm. Then, similar to the analytical approach above, it is compared whether the optimal import and export quantities reproduce the bounds of the trade algorithm. All experiments without the externality pass this test.

Calling $\pi_i pi^j_{i,k}$ the importers' i willingness to pay (WTP), and $\pi_k pe^j_{k,i}$ the exporters' k willingness to accept (WTA), the above results imply that WTP and WTA converge during iteration (if the whole algorithm converges), giving the notion of world market prices a clear meaning. Figure 3 shows an example for convergence of both. Results are based on numerical experiments with a two-region setting as analyzed in the next section.

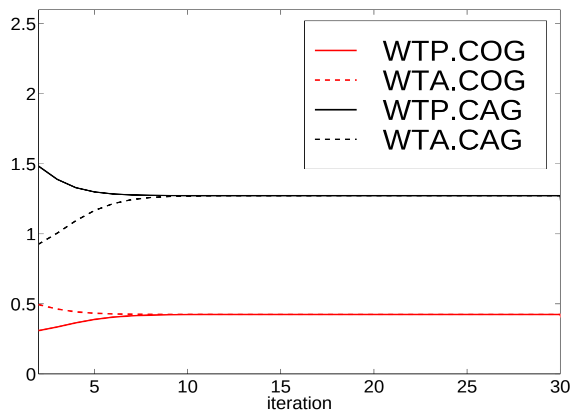

Figure 3: Convergence of the willingness to pay (WTP) and the willingness to accept (WTA) for a selected time period (COG: consumption good, CAG: capital good)

However, the above conclusion on achieving an equilibrium solution is only reliable for the non-spillover case. Spillover externalities (eq. 10) of the analyzed type generate increasing returns to scale associated with non-convexities and thus causing multiple local optima. In particular, the asymmetry of spillover effects, which occur if a region imports investment goods from a region with higher level of productivity, prevents convergence of the willingness to accept and the willingness to pay.

To represent this formally, spillover effects lead to another shadow price μ_i for region i which sums future utility gains of improved productivity due to imported capital goods. Then, the resulting prices are

$$pe_{i,k}^{F} = \lambda_i \tag{36}$$

and

$$pi_{k,i}^{F} = \lambda_i + \mu_i \frac{\partial \dot{A}_i}{\partial X_{k,i}^{F}}, \tag{37}$$

where $\mu_i > 0$ if region i imports from region k with a higher productivity level. $\dot{A}_i$ represents the productivity change (cf. eq. 10). As long as the productivity gradient remains and $\mu_i > 0$, the utility gains from import (and therefore the WTP) are always higher than from exports (and therefore the WTA). If the world market price is between the the WTA exports and the WTP for imports, we come up with a paradox situation where the region has to pay less than possible for demand, but get more than wanted for sales. It thus aims at selling all capital it has, and at the same time imports all capital that is available on the market. If there is a second region in this situation, it is unclear how a market mechanism can resolve the conflict of allocating all available capital. It is, moreover, impossible to speak of an equilibrium price if regions want to sell and buy for different prices. In our model, this problem is avoided by constraint (21) that requires each region to choose between being an exporter or an importer.

However, it remains open whether under this constraint the trade algorithm with positive spillover externalities produces as competitive equilibrium. We again performed the above mentioned numerical test. The test fails in a first instance due to the above reasons. When the externality is internalized within the decision-making process (Social Planner mode), the equivalence between the solution of the decentralized model and the Social Planner solution disappears. The decentralized agents do not behave socially optimal. A region which imports capital and receives the technological spillover under the trade algorithm, may now use the high price for capital goods to export capital goods by its own and forgoing the technological spillovers. This could increase its welfare, but it implies a trade structure that is certainly not optimal from the other regions' point of view. As suggested by economic theory, a tax or subsidy is needed to achieve the social pareto-optimum in a decentralized model. We do this by defining a market price $\tilde{p}^F$ as the weighted average of the export prices pe^F only (compare eq. 26):

$$\tilde{p}^F = \frac{\sum_{i=1}^{n}\sum_{k=1}^{n} pe_{ik}^F \cdot X_{ik}^F}{\sum_{i=1}^{n}\sum_{k=1}^{n} X_{ik}^F}\ , \qquad i \neq k. \tag{38}$$

In addition, we define a subsidy σ_{ik} that represents a mark-up on the price of capital goods exported by the technologically advanced regions. For the importing regions, this represents a tax. The intertemporal budget constraint that the decentralized agents have to meet changes to:

$$\int_{t=1}^{T}\left[p^G \cdot \sum_{k=1}^{n}(X_{ik}^G - X_{ki}^G) + (\tilde{p}^F + \sigma_{ik}) \cdot \sum_{k=1}^{n}(X_{ik}^F - X_{ki}^F)\right] dt = 0. \tag{39}$$

It is beyond the scope of this paper to find a general closed form expression for the optimal subsidy. For a 2-region setting (for which we made numerical experiments), however, we can provide an optimal σ:

$$\sigma_{i,k} = \begin{cases} p^F - \tilde{p}^F & : \quad A_i < A_k \\ 0 & : \quad A_i \geq A_k. \end{cases} \tag{40}$$

With this approach, the spillover experiments pass the above numerical optimality test. This supports the extended scope of application of the trade algorithm.

5 Model experiments

In this section, we apply the model introduced in section 3 and the developed trade algorithm in order to analyze the impact of technological spillovers on economic growth and the trade structure. We run the model within a stylized conventional 2 goods x 2 factors x 2 regions setting[4]. All results can be interpreted in a qualitative sense only. Trade in the two commodities is initiated by one of the following regional differences: factor endowments, technology (productivity), and preferences.

Technological spillovers are subject of rational expectations. The focus is on spillovers that increase the productivity in the consumption goods sector. Hence, we analyze a setting where both regions differ in their productivity level A.

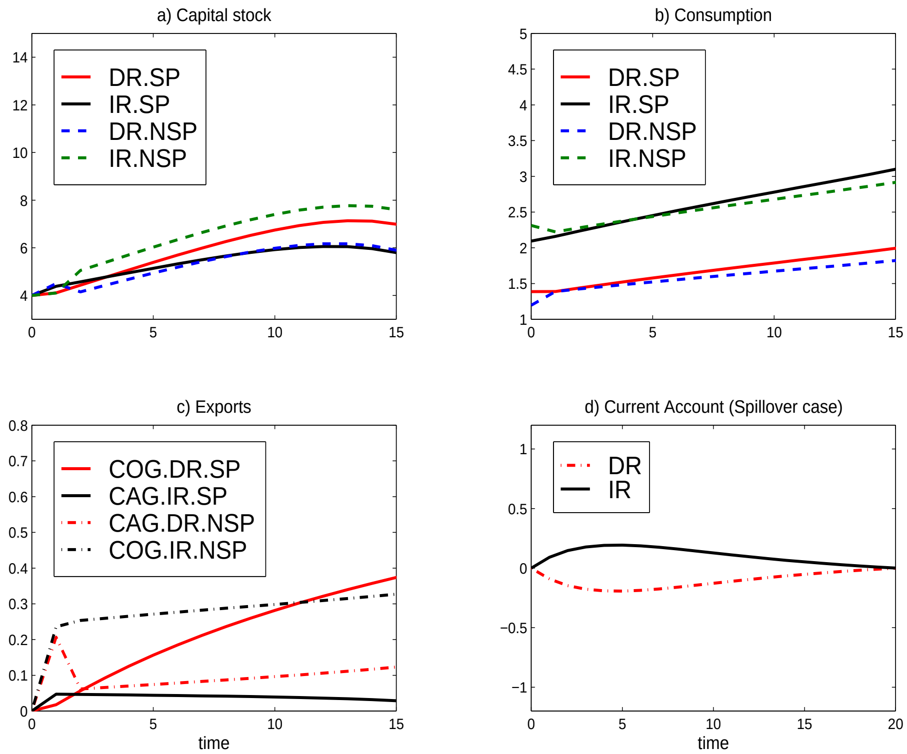

Figure 4: Impact of technological spillovers (SP: spillover case, NSP: non-spillover case, COG: consumption goods, CAG: capital goods)

¿From an economic point of view, the question arises whether there are welfare gains for both regions and what changes result in the trade structure. Figure 4 illustrates some of the impacts [5]. For presentation reasons, the two regions are labeled by IR and DR in the Figures. Under a traditional free trade scenario, region IR,

[4] All modules are programmed in GAMS (www.gams.com) and numerically solved with the nonlinear programming solver CONOPT3. The programs are available from the authors upon request. For the default parameters and initial values see Annex A.

[5] Based on eq. (11) we specify the intertemporal trade balance:

which has advantages in the production of the consumption good (higher productivity), will export the consumption good and extent its production by capital good imports. This picture changes when technological spillovers are taken into account. According to the optimal trade structure, region IR exports the investment good and region DR exports the consumption good. The dynamics of the current account is also reversed. Whereas DR starts with a trade surplus in the non-spillover case, a trade balance deficit comes with the optimal solution within the spillover scenario. The consumption path in Figure 4 indicates gains in consumption for both regions. This is further specified in Figure 5.

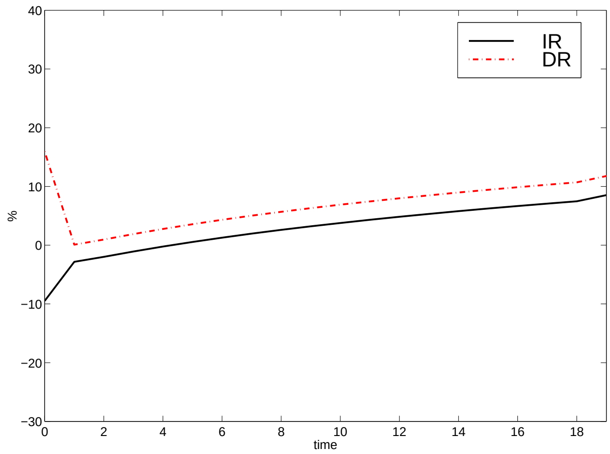

Figure 5: Per capita consumption gains from technological spillovers ($A_{IR} = 2.0$, $A_{DR} = 1.2$)

Consumption gains in both regions increase with time. There are yet significant differences in the patterns of gains. DR gains in all periods. IR, in contrast, loses in initial periods. Consumption and welfare gains of DR are directly linked to productivity increases caused by technological spillovers. Positive feedbacks to IR, while on a moderate level only, are mainly due to higher prices of investment goods. Figure 6 demonstrates the price differences between the spillover case and the non-spillover case. Differences exist for the investment goods only. Higher

---

$$TB_{it} = \sum_{\tau=1}^{t} B_{i\tau}.$$

prices of investment goods occur due to the anticipation of the spillover effect and the willingness of DR to pay a higher price. Region IR, which now becomes an exporter of investment goods, benefits from this. In order to meet the intertemporal trade balance, DR compensates the expansion of investment goods imports (compared to the non-spillover case) increasingly by consumption goods exports.

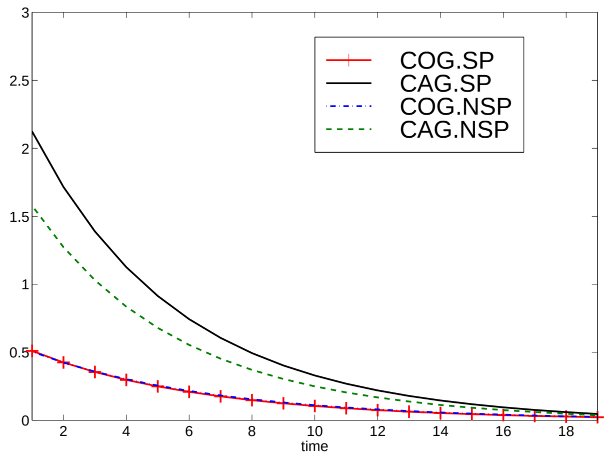

Figure 6: Price levels of tradeable goods (SP: spillover case, NSP: non-spillover case, COG: consumption goods, CAG: capital goods).

Due to the discounting effect, consumption gains of region IR do not manifest in an equal increase in welfare. While IR increases its welfare compared to the autarky case by 0.5%, it loses compared to the non-spillover case by 0.6% (DR increases welfare by 17.0%). IR faces weakened terms of trade. As IR is exporter of consumption goods in the non-spillover case (see Figure 4), it first suffers from a decrease of relative prices of consumption goods. Becoming an exporter of investment goods and benefiting from the price increase can not completely compensate this terms-of-trade effect.

However, the level of positive feedbacks from the spillover effects on the capital-exporting region heavily depends on the existing trade structure and diversity of regional characteristics. Within a scenario where IR is not only distinguished by higher productivity in the consumption goods sector but also by:

- higher productivity in the investment good sector ,
- higher capital intensity,

• lower discount rate,

IR increases its welfare compared to the non-spillover scenario (e.g. in the scenario with higher capital intensity by 0.7%). Higher capital intensity is implemented as a reduction of exogenous labor supply to 60%. Higher productivity is implemented by increasing parameter κ_{IR} from 0.16 to 0.18. For these scenarios, the change of consumption gains is demonstrated in Figure 7. In general, benefits occur for all cases where IR is an exporter of investment goods already in the non-spillover scenario. While this is accompanied by a higher growth rate of percentage consumption gains, it does not change the impact pattern of technological spillovers on consumption (as shown by Figure 5).

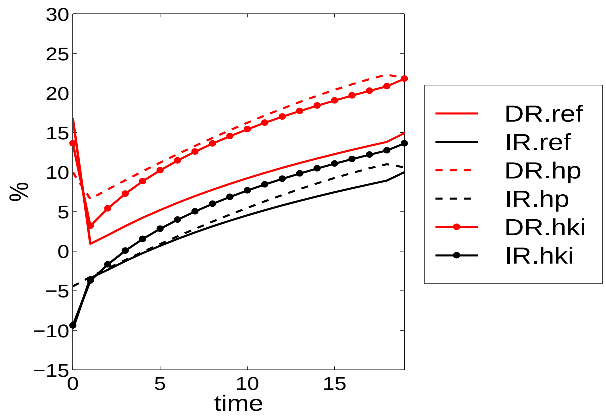

Figure 7: Per capita consumption gains from technological spillovers (ref - Spillover reference scenario, hp - higher productivity of IR in investment goods sector , hki - higher capital intensity in IR)

Due to the fact that the mathematical model structure becomes non-convex by introducing spillover effects, multiple optima may exist. The above algorithm of finding the optimal solution to the multi-region optimization problem does not guarantee that a global optimum is found. Nevertheless, the convergence process in finding the local optimum is quite robust (see Figure 2 and 3).

We carried out sensitivity analyses with respect to the external effect. The spillover coefficient β was varied within the interval [0, 1.6]. Smooth changes of welfare and per capita consumption over a wide range (see Figure 8 and Figure 9) demonstrate again robustness of the approach.

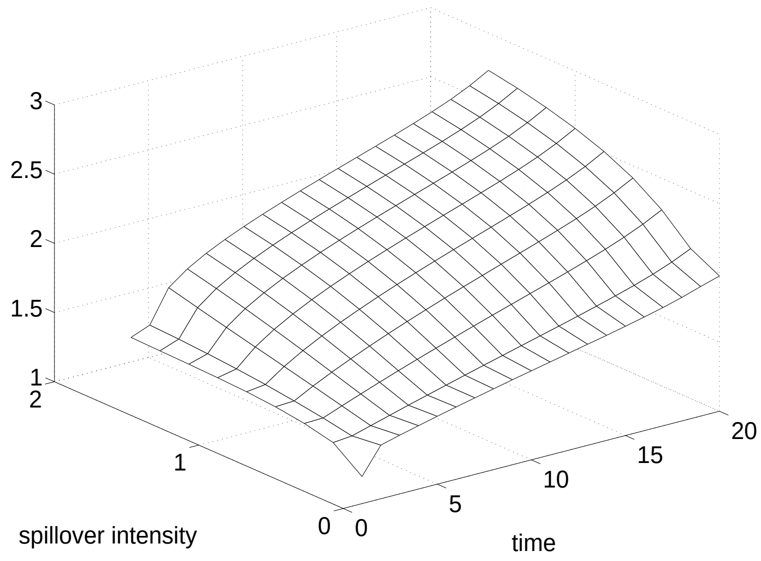

Figure 8: Per capita consumption sensitivity on spillover intensity in DR

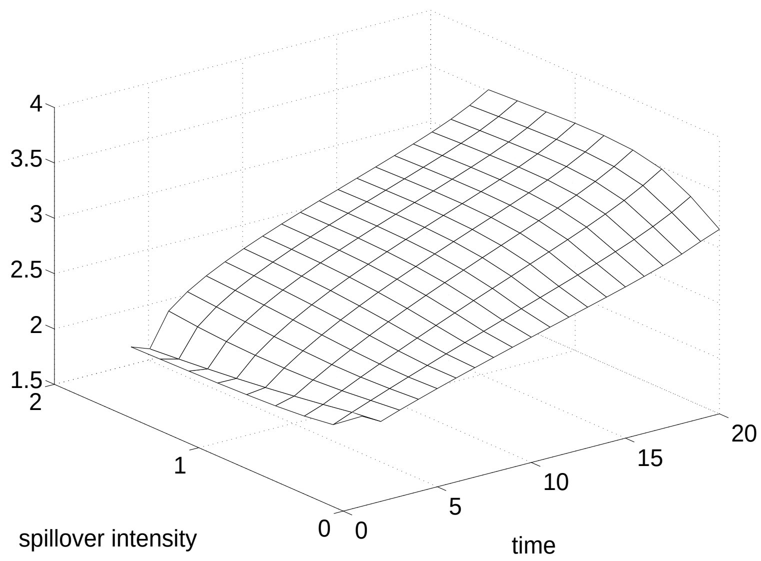

Figure 9: Per capita consumption sensitivity on spillover intensity in IR

6 Conclusions

We presented an alternative approach to multi-region modeling based on a novel trade algorithm. This approach is applicable in an intertemporal optimization framework. On the one hand, it shares some basic features with the approach of Leimbach and Edenhofer (2007), on the other hand, it includes a completely different trade module. Due to its similarity with the Negishi approach, this new approach is more transparent and convincing in the way it derives a pareto-optimal solution. This is further supported by an in-depth analysis of the analytical properties of the trade algorithm.

The algorithm presented in this paper is distinguished by its ability to deal with spillover externalities numerically. It is applicable to other types of externalities as well. First experiences exist in analyzing external effects from greenhouse gas emissions. However, further research is needed to proof general validity of the algorithm for cases with externalities.

This paper deals with technological spillovers as an effect that can be anticipated by agents when deciding on investments and exports. A socially optimal behavior of agents is assumed. Exemplary numerical experiments show that in the presence of technological spillovers, the optimal trade structure may reverse. The most significant spillover effect is the primary productivity-increasing effect for the capital importing region. Secondary price and terms of trade effects will affect the capital-exporting region as well. Benefits for that region are the higher the more it would export investment goods already in the absence of technological spillovers. This would guarantee that this region benefits completely from higher prices of the investment goods that occur in the presence of technological spillovers. Higher capital stock per capita, higher productivity in the investment goods sector and a lower pure rate of time preference increase the benefits of the spillover effect for the capital-exporting region.

Acknowledgement

We would like to thank Ottmar Edenhofer, Sylvie Marchand, Kai Lessmann and Robert Marschinski for fruitful discussions and valuable aid.

References

[1] Böhringer, C., T.F. Rutherford (2002). "Carbon Abatement and International Spillovers", Environmental and Resource Economics 22, 391-417.

[2] Corsetti, G., P. Martin, P. Pesenti (2005). "Productivity Spillovers, Terms of Trade and the 'Home Market Effect'", Discussion Paper No. 4964, Centre for Economic Policy Research, London.

[3] Farmer, R., A. Lahiri (2005). "A two-country model of endogenous growth", Review of Economic Dynamics 8, 68-88.

[4] Ginsburgh, V.A., J.L. Waelbroeck (1981). Activity Analysis and General Equilibrium Modelling. Amsterdam: North-Holland.

[5] Greiner, A., W. Semmler (2002). "Externalities of investment, education and economic growth", Economic Modelling 19, 709-724.

[6] Jordaan, A.J. (2005). "Determinants of FDI-Induced Externalities: New Empirical Evidence for Mexican Manufaturing Industries", World Development 33, 2103-2118.

[7] Kokko, A. (1993). Foreign Direct Investments, Host Country Characteristics, and Spillover. Dissertation, EFI/The Economic Research Institute, Stockholm School of Economics, Stockholm.

[8] Kumar, A., M. Shubik (2004). "Variations on the Theme of Scarf's Counter-Example", Computational Economics 24, 1-19.

[9] Lee, J.-W. (1995). "Capital goods import and long-run growth", Journal of Development Economics 48, 91-110.

[10] Leimbach, M., F.L. Toth (2003). "Economic Development and Emission Control over the Long Term: The ICLIPS Aggregated Economic Model", Climatic Change 56, 139-165.

[11] Leimbach, M., O. Edenhofer (2007). "Technological spillovers within multi-region models: Intertertemporal optimization beyond the Negishi approach", Economic Modelling, in press.

[12] Luenberger, D.G., R.R. Maxfield (1995). "Computing Economic Equilibria Using Benefit and Surplus Functions", Computational Economics 8, 47-64.

[13] Manne, A.S., M. Mendelsohn, R. Richels (1995). "MERGE - A Model for Evaluating Regional and Global Effects of GHG Reduction Policies", Energy Policy 23, 17-34.

[14] Manne, A.S., R. Richels (1995). "The Greenhouse Debate: Economic Efficiency, Burden Sharing and Hedging Strategies", The Energy Journal 16, 1-37.

[15] Manne, A.S., T.F. Rutherford (1994). "International Trade, Capital Flows and Sectoral Analysis: Formulation and Solution of Intertemporal Equilibrium Models." In: Cooper, W.W., A.B. Whinston (eds.), New Directions in Computational Economics. Dordrecht: Kluwer Academic Publishers, pp. 191-205.

[16] Oniki, H., H. Uzawa (1965). "Patterns of Trade and Investment in a Dynamic Model of International Trade", Review of Economic Studies 32, 15-38.

[17] Negishi, T. (1972). General Equilibrium Theory and International Trade. Amsterdam: North-Holland.

[18] Nordhaus, W.D., Z. Yang (1996). "A Regional Dynamic General-Equilibrium Model of Alternative Climate-Change Strategies", American Economic Review 86, 741-765.

[19] Samuelson, P.A. (1952). "Spatial Price Equilibrium and Linear Programming", American Economic Review 42, 283-303.

[20] Scarf, H. (1984). "The Computation of Equilibrium Prices." In: Scarf, H., J. Shoven (eds.), Applied General Equilibrium Analysis. Cambridge: Cambridge University Press.

[21] Stiglitz, J.E. (1970). "Factor Price Equalization in a Dynamic Economy", The Journal of Political Economy 78, 456-488.

[22] Takii, S. (2004). "Productivity Differentials Between Local and Foreign Plants in Indonesian Manufacturing, 1995", World Development 32, 1957-1969.

A Analytical properties of the trade algorithm

It is shown in this appendix that the trade algorithm produces a competitive equilibrium if it converges. As by-product other properties of this fixed point become clear. The strategy is as follows. At first we derive some analytical features of the regional module (for given import- and export bounds) and of the trade module (for given welfare weights), assuming an interior solution. When the algorithm converges, both modules compute coherent results. We show that these results respect the first order conditions of the non-decomposed basic model. For clarity, all variables of the trade module are indicated by a bar $\bar{\cdot}$, those of the basic model by a hat $\hat{\cdot}$, while variables of the regional module have no particular accents.

A.1 Regional module for the case without spillovers

We first derive the optimal allocation paths θ_i of the regions i in the regional module, where import and export bounds $\bar{X}_i^j$ are prescribed by the trade module. Therefore, the latter are parameters in the optimization problems of the regional module. The derivative of optimal utility with respect to these paramters is used by the trade module for the next iteration (cf. Eqs. 27,28). Since we concentrate on interior solutions in the sense that $I_i > 0$, the current-value Hamiltonian L_i for the regional module reads

$$L_i = \ln(C_i) + \lambda_i(Y_i^F - \Delta\bar{X}_i^F - \delta_i K_i), \tag{41}$$

where, for convenience, net exports of region i are denoted by

$$\Delta\bar{X}_i^j := \sum_k (\bar{X}_{i,k}^j - \bar{X}_{k,i}^j). \tag{42}$$

Note that for the condition $C_i \geq 0$ no Kuhn-Tucker parameter is needed due to the logarithmic form of the utility function. The costate variable λ_i represents the shadow price for capital. The associated equation of motion for the shadow price is

$$\dot{\lambda}_i = \lambda_i(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{1}{C_i}\frac{\partial Y_i^G}{\partial K_i}, \tag{43}$$

together with the transversality condition $\lambda(T) = 0$.

At each time step, the region chooses the capital allocation θ_i such that L_i is maximized. This amounts in solving

$$-\frac{\partial Y_i^G}{\partial \theta_i}\frac{1}{C_i} = \lambda_i \frac{\partial Y_i^F}{\partial \theta_i} \tag{44}$$

for θ_i.

By the envelope theorem, the optimal utility U_i^* depends on the paramters $\bar{X}$ by $\frac{\partial U_i^*}{\partial \bar{X}} = \frac{\partial L_i^*}{\partial \bar{X}}$, if the latter is evaluated on the optimal production path. Therefore,

$$-\frac{\partial U_i^*}{\partial \bar{X}_{i,k}^G} = pe_{i,k}^G = -\frac{1}{C_i}, \tag{45}$$

$$-\frac{\partial U_i^*}{\partial \bar{X}_{i,k}^F} = pe_{i,k}^F = -\lambda_i. \tag{46}$$

A.2 Trade module

For the trade module, the Hamiltonian is expanded to

$$L = \sum_i w_i \ln(\bar{C}_i) + \sum_i \bar{\lambda}_i (Y_i^F - \Delta \bar{X}_i^F - \delta_i \bar{K}_i). \tag{47}$$

Based on the costate equation

$$\dot{\bar{\lambda}}_i = \bar{\lambda}_i(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{\bar{C}_i}\frac{\partial Y_i^G}{\partial K_i}, \tag{48}$$

with the transversality condition $\bar{\lambda}_i(T) = 0$, the trade bounds are computed from $\max_{\bar{X}_{k,i}^j, \bar{X}_{i,k}^j, \bar{\theta}_i} L$. Hence, the optimal allocation $\bar{\theta}_i$ is determined from

$$-\frac{\partial Y_i^G}{\partial \theta_i}\frac{w_i}{\bar{C}_i} = \bar{\lambda}_i \frac{\partial Y_i^F}{\partial \theta_i}. \tag{49}$$

The derivatives of the Hamiltonian with respect to export and import bounds are

$$\frac{\partial L}{\partial \bar{X}_{i,k}^G} = -\frac{\partial L}{\partial \bar{X}_{k,i}^G} = \frac{w_k}{\bar{C}_k} - \frac{w_i}{\bar{C}_i}, \tag{50}$$

and

$$\frac{\partial L}{\partial \bar{X}_{i,k}^F} = -\frac{\partial L}{\partial \bar{X}_{k,i}^F} = \bar{\lambda}_k - \bar{\lambda}_i. \tag{51}$$

Setting Eq. (50) to zero describes an equation system for all net exports for the consumption good, since $\bar{C}_i = Y_i^G - \Delta \bar{X}_i^G$ by definition. From its solution, import and export quantities are uniquely determined by the constraint Eq. (22). We can also derive the equation

$$\forall i, k = 1, \ldots, n : \frac{\bar{C}_i}{\bar{C}_k} = \frac{w_i}{w_k}. \tag{52}$$

For the trade of investment goods the situation is more complicated, since Eq. (51) does not depend directly on $\bar{X}^F_{i,k}$. In the steady-state solution capital changes $\dot{\bar{K}}_i$ are chosen such that

$$\forall i, k = 1, \ldots, n : \bar{\lambda}_i \equiv \bar{\lambda}_k. \tag{53}$$

In the following, we concentrate on that case, justifying to introduce the variable $\bar{\lambda} := \bar{\lambda}_i = \bar{\lambda}_k$ for the solution of the costate equation.

We now put together the results for the regional and trade module in the fixed point of the algorithm. There, $\theta_i = \bar{\theta}_i, C_i = \bar{C}_i$, and the intertemporal budget is balanced. Comparing the costate equations (43) and (48), we now prove the important equation

$$\forall i = 1, \ldots, n : \bar{\lambda} \equiv w_i \lambda_i. \tag{54}$$

Define $\bar{\lambda} = w_i \lambda_i$. Then, by Eq. (43)

$$\dot{\bar{\lambda}}_i = w_i \dot{\lambda}_i = w_i \lambda_i (\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{C_i} \frac{\partial Y_i^G}{\partial K_i}, \tag{55}$$

which in the fixed point equals (by definition of $\bar{\lambda}$)

$$\bar{\lambda}(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{w_i}{\bar{C}_i} \frac{\partial Y_i^G}{\partial K_i}, \tag{56}$$

which obviously solves Eq. (48). Trivially, $\bar{\lambda}(T) = 0 = w_i \lambda_i(0)$. Therefore, $\bar{\lambda}$ is indeed the shadow price computed by the trade module. This shows that the shadow prices for capital in the regions only differ by a constant coefficient which is equivalent to its welfare weight.

A.3 Basic model

We will now show that the trade algorithm computes an international competitive equilibrium if it converges. This is the case when the optimal trade flows of the decentralized basic model reproduce the results of the trade algorithm. We demonstrate this by validating that the dynamics induced by the solution of the algorithm, namely

$$\hat{\theta}_i = \theta_i, \quad \hat{X}_i^j = \bar{X}_i^j, \tag{57}$$

and consequently $\hat{C}_i = C_i$, is consistent with the first order conditions of the basic model. Again, we concentrate on interior solutions.

The Hamiltonian of the basic model is

$$L_i = \ln(\hat{C}_i) + \hat{\lambda}_i(Y_i^F - \Delta\hat{X}_i^F - \delta\hat{K}_i) + \hat{\pi}_i(p^F\Delta\hat{X}_i^F + p^G\Delta\hat{X}_i^G). \qquad (58)$$

The new costate variable $\hat{\pi}_i$ corresponds to the budget balance equation (cf. Eq. 11)

$$\dot{\hat{B}}_i = p^F(t)\Delta\hat{X}_i^F(t) + p^G(t)\Delta\hat{X}_k^G(t), \qquad (59)$$

and $B_i(T) = 0$. Note that the latter is always satisfied if quantities and prices are determined from the convergent trade module. We assume that world market prices are determined by $p^F(t) = e^{-\rho t}\frac{1}{n}\sum_k w_k\lambda_k(t)$ and $p^G(t) = e^{-\rho t}\frac{1}{n}\sum_k \frac{w_k}{C_k}$ in the fixed point of the trade algorithm. The associated costate equation is solved by

$$\hat{\pi}_i(t) = \hat{\pi}_i(0)e^{\rho t}, \qquad (60)$$

where $\hat{\pi}_i(0)$ has to be chosen appropriately. The costate equation for $\hat{\lambda}$ evolves according to

$$\dot{\hat{\lambda}}_i = \hat{\lambda}_i(\rho + \delta_i - \frac{\partial Y_i^F}{\partial K_i}) - \frac{1}{\hat{C}_i}\frac{\partial Y_i^G}{\partial K_i}, \qquad (61)$$

and $\hat{\lambda}_i(T) = 0$, which is identical to the costate equation for the regional module (see Eq. 43). Note that, due to convergence, also $\hat{C}_i = \bar{C}_i$, (cf. eq. 31) such that we conclude

$$\hat{\lambda}_i \equiv \lambda_i. \qquad (62)$$

So far we have only determined how the state and costate equations of the basic model evolve if the control variables are chosen as in the result of the trade algorithm. Now it has to be determined whether the control variables also maximize the Hamiltonian of the basic model. We claim that choosing

$$\hat{\pi}_i(0) = w_i^{-1} \qquad (63)$$

is appropriate. This is an intertemporal analogue to the Negishi approach, where the inverse of Negishi weights is equal to the marginal utilities of income.

By differentiating the Hamiltonian with respect to the control variable $\hat{\theta}_i$, we obtain the first order condition

$$0 = \hat{\lambda}_i\frac{\partial Y_i^F}{\partial\theta_i} - \frac{\partial Y_i^G}{\partial\theta_i}\frac{1}{\hat{C}_i} = \lambda_i\frac{\partial Y_i^F}{\partial\theta_i} - \frac{\partial Y_i^G}{\partial\theta_i}\frac{1}{C_i}, \qquad (64)$$

where the second equation is due to Eq. (62) and $\hat{C}_i = C_i$. This equation is obviously the same as Eq. (49), such that its solutions is indeed identical to θ_i. For import and export quantities, the first order conditions are

$$\frac{\partial L}{\partial \hat{X}^G_{i,k}} = -\frac{\partial L}{\partial \hat{X}^G_{k,i}} = \hat{\pi}_i p^G - \frac{1}{\hat{C}_i} = 0, \tag{65}$$

and

$$\frac{\partial L}{\partial \hat{X}^F_{i,k}} = -\frac{\partial L}{\partial \hat{X}^F_{k,i}} = \hat{\pi}_i p^F - \hat{\lambda}_i = 0. \tag{66}$$

Due to Eq. (52),

$$\hat{\pi}_i p^F - \hat{\lambda}_i = \hat{\pi}_i e^{-\rho t} \frac{1}{n} \sum_k \frac{w_k}{C_k} - \frac{1}{C_i} = \hat{\pi}_i(0) \frac{w_i}{C_i} - \frac{1}{C_i}. \tag{67}$$

Since $\pi_i(0) w_i = 1$, the first order condition for trade with the consumption good is satisfied. By additionally using Eq. (62) we also obtain

$$\hat{\pi}_i p^F - \hat{\lambda}_i = \hat{\pi}_i e^{-\rho t} \frac{1}{n} \sum_j w_j \lambda_j - \lambda_i \tag{68}$$

for the capital good. This is, by Eq. (54) equivalent to

$$\hat{\pi}_i(0) \frac{1}{n} \sum_j w_j \frac{\tilde{\lambda}}{w_j} - \frac{\tilde{\lambda}}{w_i} = \hat{\pi}_i(0) \tilde{\lambda} - \frac{\tilde{\lambda}}{w_i}, \tag{69}$$

and vanishes again due to $\pi_i(0) w_i = 1$. We can thus conclude that the dynamics of the basic model induced by the trade structure of the trade algorithm is consistent with an optimal selection of control variables in the decentralized case.

As a by-product we have derived an interpretation for the welfare weights. As one corrolary, the regional shadow prices for capital λ_i differ from the world market price only by the ratio $\hat{\pi}_i$, which is proportional to w_i^{-1}.

B Default parameters and initial values

ρ	:	0.03
δ	:	0.08
β	:	0.4
ζ	:	0.4
α_{IR}	:	0.33
α_{DR}	:	0.33
κ_{IR}	:	0.16
κ_{DR}	:	0.16
ϕ	:	0.9
k_{IR}	:	4
k_{DR}	:	4
a_{IR}	:	1.2
a_{DR}	:	1.2
$L_{IR}(0)$	:	1.0 (constant)
$L_{DR}(0)$	:	1.0 (constant)
$\bar{X}_1^j$	:	0.0
w_1	:	1.0